Advanced Math

1-6
Inverse Functions

Inverse of a function

Blue Collar Definition - Two functions are inverses if
they cancel each other.

Graphical Definition - Two functions are inverses if
$(x,y) \rightarrow (y,x)$; domain + range are switched.
reflections about $y=x$

Mathematician's Definition - Two functions $f(x)$ and $g(x)$
are inverses iff (if and only if)

a) $f(g(x)) = x$

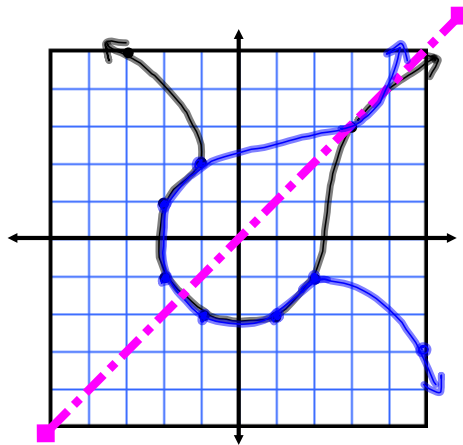b) $g(f(x)) = x$

Show that f and g are inverse functions.

*) $f(x) = 5 - 4x$ $\qquad\qquad g(x) = \dfrac{5 - x}{4}$

a) $F(g(x)) = F\left(\frac{5-x}{4}\right) = 5 - 4\left(\frac{5-x}{4}\right)$

$\qquad\qquad\qquad = 5 - (5-x) = 5 - 5 + x = \boxed{x}\ \checkmark$

b) $g(F(x)) = g(5-4x) = \dfrac{5-(5-4x)}{4} = \dfrac{5-5+4x}{4} = \dfrac{4x}{4} = \boxed{x}\ \checkmark$

Yes, inverses! □

Sketch the inverse of each graph. Is the inverse a function?

one-to-one function - A Function where each output corresponds with exactly one input.

Not one to one.

One to one functions will always have inverses that are also functions.

~~Determine whether the function has an inverse. If it does, find it.~~
(Determine whether the inverse is a function or not. Find it regardless.)

49) $f(x) = (x + 3)^2$ $f^{-1}(x) = -3 \pm \sqrt{x}$

$$y = (x+3)^2$$

inverse

Switch y and x. Graphical definition of inverse. Then get y itself.

$$\sqrt{x} = \sqrt{(y+3)^2}$$
$$\sqrt{x} = |y+3|$$
$$\pm\sqrt{x} = y+3$$
$$-3 \pm \sqrt{x} = y$$

Use these instructions instead of the books.

inverse is not a Function

Two outputs for each input, therefore not 1 to 1.

Use the functions $f(x) = x + 4$ and $g(x) = 2x - 5$ to find the following:

75) $g^{-1} \circ f^{-1}$

$$g^{-1}(f^{-1}(x))$$

$$g^{-1}(x-4) =$$

$$= \frac{(x-4)+5}{2}$$

$$\boxed{g^{-1}(f^{-1}(x)) = \frac{x+1}{2}}$$

$f(x) = x+4$
$y = x+4$
$x = y+4$
$x-4 = y$
$f^{-1}(x) = x-4$

$g(x) = 2x-5$
$x = 2y-5$
$x+5 = 2y$
$\frac{x+5}{2} = y$

$\frac{x+5}{2} = g^{-1}(x)$

Assignment:

pg. 180
1 - 4,
12 - 58 even,
76, 78, 81